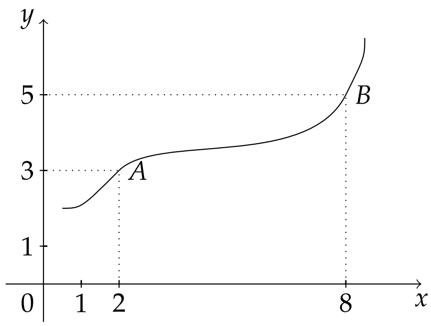
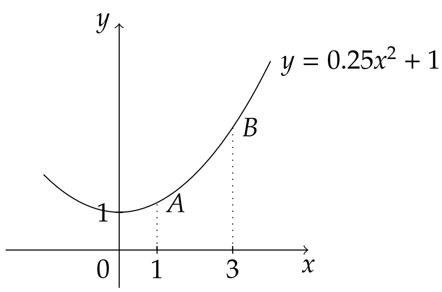
Exercises in differentiation

1. Find the gradient of the secant of the curve $y = x^3 - 3x^2 + x + 2$ passing through the points $A = (x_A, y_A)$ and $B = (x_B, y_B)$, knowing that $x_A = 1$ and $y_B = 5$.

2. Find the average rate of change from the point A to the point B.

(a) 

(b) 

(c) $y = -2x^2 + \sqrt{x}, \quad x_A = 1, \quad x_B = 2;$

(d) $g(t) = \frac{2t-1}{\sqrt{t^2+1}}, \quad t_A = -1, \quad t_B = 1.$

3. Find the gradient of the secant of the curve $y = x^3 - 3x^2 + x + 2$ passing through the points $A = (x_A, y_A)$ and $B = (x_B, y_B)$, knowing that $x_A = 1$ and $y_B = 5$.

4. Compute the average rate of change of the function $f(x) = \frac{x^3-2x}{x}$ from some x_0 to $x_0 + h$, where $h \neq 0$. Use a limit argument to find $\frac{df}{dx}(x_0)$.

5. Find the derivatives of the following functions at the points given:

(a) $f(x) = 3x^2 + 5x - 8, \quad x_0 = -2;$

(b) $g(x) = \frac{1}{x^2} + 2, \quad x_0 = 1;$

(c) $y(x) = \sqrt{x} + \sqrt[3]{x}, \quad x_0 = 64;$

(d) $x(t) = 5 + \frac{t^2-1/t}{t}, \quad t_0 = 3.2;$

(e) $u(s) = 1 + s + s^2 + s^3, \quad s_0 = -1;$

(f) $p(q) = (100 - 3q^2)/q, \quad q_0 = 8;$

(g) $y(a) = a^2 - 3a + 2, \quad a_1 = 1, \quad a_2 = 1.5, \quad a_3 = 2;$

(h) $x(t) = (t-3)^2, \quad t_1 = -3, \quad t_2 = 0, \quad t_3 = 3;$

(i) $f(x) = 3\sqrt[4]{x}, \quad x_1 = 2.8, \quad x_2 = 100, \quad x_3 = 0.023;$

(j) $\gamma(p) = (p^2+1)(p-1)^2, \quad p_1 = 1, \quad p_2 = 10, \quad p_3 = 100;$

(k) $s(t) = t^{10} - \frac{7}{t^{14}} + 5\sqrt{7}, \qquad t_0 = 1;$

(l) $x(y) = \frac{y^3 - y}{y^2}, \qquad y_0 = -1;$

(m) $f(u) = \sqrt{u} + \sqrt[3]{u} - \sqrt[5]{u}, \qquad u_0 = 3.2;$

(n) $v(z) = \frac{(z-1)^2}{z^2}, \qquad z_0 = 1;$

(o) $g(x) = \frac{\sqrt{x}+1}{\sqrt{x}}, \qquad x_0 = 8;$

(p) $y(x) = (x^2 + x + 1)(x - 1) - x^3, \qquad x_0 = 2007.$

6. Differentiate the following functions:

(i) $h(y) = y^{100} - 1/y^{100};$

(ii) $\alpha(t) = t - \frac{t^3}{6} + \frac{t^5}{120} - \frac{t^7}{5040};$

(iii) $\phi(r) = \sqrt[4]{r} + \frac{1}{r^4} + 4;$

(iv) $a(u) = \frac{1}{2}(u^2 - u^{-2};$

(v) $x(y) = \frac{y^2(y-1)}{3y^3};$

(vi) $M(x) = \frac{1}{\sqrt{x}}(\sqrt{x} + x + x^2) - \frac{1}{x}(x + x\sqrt{x} + x^{2.5});$

(vii) $f(x) = -2x\sqrt{x};$

(viii) $f(x) = \frac{3}{x^2};$

(ix) $f(x) = -\frac{8}{x^8};$

(x) $f(x) = 3x^2\sqrt{x};$

(xi) $x(t) = 3t^2\sqrt{t^3};$

(xii) $p(q) = -\frac{qx}{\sqrt[3]{q^2}};$

(xiii) $f(x) = x - \frac{x^3}{6} + x^5 120;$

(xiv) $f(x) = 1 + x^2 + x^4;$

(xv) $m(l) = l\sqrt{l} + 1/\sqrt{l};$

(xvi) $q(p) = 200 + \frac{500}{p};$

(xvii) $y(\theta) = (\theta + 1)(\theta - 1);$

(xviii) $p(\tau) = (\tau^2 - 1)(\tau - 3);$

(xix) $f(x) = 5;$

(xx) $f(x) = \frac{3}{x^2};$

(xxi) $f(x) = -3;$

(xxii) $f(x) = 2\pi x;$

(xxiii) $g(y) = y^2\sqrt{2};$

(xxiv) $h(k) = \frac{2}{\sqrt{3k}};$

(xxv) $f(x) = x^3 + x^2;$

(xxvi) $f(x) = 3x - \frac{2}{x};$

(xxvii) $f(x) = x^3 - \frac{1}{x^3};$

(xxviii) $g(y) = 4y^2 + 5y^5 - \frac{2}{y^2};$

(xxix) $x(t) = t^2 + 4t + 4;$

(xxx) $s(t) = (t + 2)^2.$

7. Find the equation of the tangent to the graph of $f(x)$ at x_0.

(a) $f(x) = x^2 + 2x + 3, \qquad x_0 = -1;$

(b) $f(x) = \frac{1}{3}[x(\sqrt{x} - 1) - x^3], \qquad x_0 = 4;$

(c) $f(x) = \frac{x^3 - \frac{1}{\sqrt{x}} + 2}{\sqrt{x}}, \qquad x_0 = 3.1.$

8. A particle is moving along the x axis in such a way that its position (in cm) after t seconds is given by the formula $x(t) = 3 + 2t - \frac{1}{2}t^2$.

(a) Find the initial position of the particle (i.e., the position in the moment $t = 0$) and its position after 3 s.

(b) At which moment will the particle reach the origin of the coordinate system?

(c) Find the formula for the velocity v after t seconds. [*Hint*: differentiate $x(t)$.]

(d) What is the initial velocity? What is the velocity after 3 s? Does your result make sense?

(e) Prove that the motion is uniformly retarded, i.e., the acceleration is negative and constant.