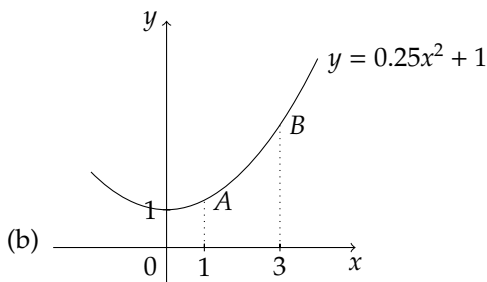
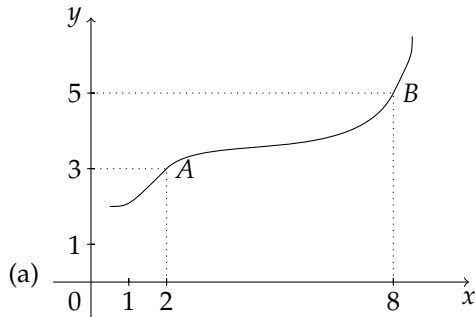


Exercises in differentiation

- Find the gradient of the secant of the curve $y = x^3 - 3x^2 + x + 2$ passing through the points $A = (x_A, y_A)$ and $B = (x_B, y_B)$, knowing that $x_A = 1$ and $y_B = 5$.
- Find the average rate of change from the point A to the point B .



- (c) $y = -2x^2 + \sqrt{x}$, $x_A = 1$, $x_B = 2$;
 (d) $g(t) = \frac{2t-1}{\sqrt{t^2+1}}$, $t_A = -1$, $t_B = 1$.
- Find the gradient of the secant of the curve $y = x^3 - 3x^2 + x + 2$ passing through the points $A = (x_A, y_A)$ and $B = (x_B, y_B)$, knowing that $x_A = 1$ and $y_B = 5$.
 - Compute the average rate of change of the function $f(x) = \frac{x^3-2x}{x}$ from some x_0 to $x_0 + h$, where $h \neq 0$. Use a limit argument to find $\frac{df}{dx}(x_0)$.
 - Find the derivatives of the following functions at the points given:
 - $f(x) = 3x^2 + 5x - 8$, $x_0 = -2$;
 - $g(x) = \frac{1}{x^2} + 2$, $x_0 = 1$;
 - $y(x) = \sqrt{x} + \sqrt[3]{x}$, $x_0 = 64$;
 - $x(t) = 5 + \frac{t^2-1}{t}$, $t_0 = 3.2$;
 - $u(s) = 1 + s + s^2 + s^3$, $s_0 = -1$;
 - $p(q) = (100 - 3q^2)/q$, $q_0 = 8$;
 - $y(a) = a^2 - 3a + 2$, $a_1 = 1$, $a_2 = 1.5$, $a_3 = 2$;
 - $x(t) = (t - 3)^2$, $t_1 = -3$, $t_2 = 0$, $t_3 = 3$;
 - $f(x) = 3\sqrt[4]{x}$, $x_1 = 2.8$, $x_2 = 100$, $x_3 = 0.023$;
 - $\gamma(p) = (p^2 + 1)(p - 1)^2$, $p_1 = 1$, $p_2 = 10$, $p_3 = 100$;

- (k) $s(t) = t^{10} - \frac{7}{t^{14}} + 5\sqrt{7}$, $t_0 = 1$;
 (l) $x(y) = \frac{y^3 - y}{y^2}$, $y_0 = -1$;
 (m) $f(u) = \sqrt{u} + \sqrt[3]{u} - \sqrt[5]{u}$, $u_0 = 3.2$;
 (n) $v(z) = \frac{(z-1)^2}{z^2}$, $z_0 = 1$;
 (o) $g(x) = \frac{\sqrt{x+1}}{\sqrt{x}}$, $x_0 = 8$;
 (p) $y(x) = (x^2 + x + 1)(x - 1) - x^3$, $x_0 = 2007$.

6. Differentiate the following functions:

- (i) $h(y) = y^{100} - 1/y^{100}$;
 (ii) $\alpha(t) = t - \frac{t^3}{6} + \frac{t^5}{120} - \frac{t^7}{5040}$;
 (iii) $\phi(r) = \sqrt[4]{r} + \frac{1}{r^4} + 4$;
 (iv) $a(u) = \frac{1}{2}(u^2 - u^{-2})$;
 (v) $x(y) = \frac{y^2(y-1)}{3y^3}$;
 (vi) $M(x) = \frac{1}{\sqrt{x}}(\sqrt{x} + x + x^2) - \frac{1}{x}(x + x\sqrt{x} + x^{2.5})$;
 (vii) $f(x) = -2x\sqrt{x}$;
 (viii) $f(x) = \frac{3}{x^2}$;
 (ix) $f(x) = -\frac{8}{x^8}$;
 (x) $f(x) = 3x^2\sqrt{x}$;
 (xi) $x(t) = 3t^2\sqrt{t^3}$;
 (xii) $p(q) = -\frac{qx}{\sqrt[3]{q^2}}$;
 (xiii) $f(x) = x - \frac{x^3}{6} + x^5 120$;
 (xiv) $f(x) = 1 + x^2 + x^4$;
 (xv) $m(l) = l\sqrt{l} + 1/\sqrt{l}$;
 (xvi) $q(p) = 200 + \frac{500}{p}$;
 (xvii) $y(\theta) = (\theta + 1)(\theta - 1)$;
 (xviii) $p(\tau) = (\tau^2 - 1)(\tau - 3)$;
 (xix) $f(x) = 5$;
 (xx) $f(x) = \frac{3}{x^2}$;
 (xxi) $f(x) = -3$;
 (xxii) $f(x) = 2\pi x$;
 (xxiii) $g(y) = y^2\sqrt{2}$;
 (xxiv) $h(k) = \frac{2}{\sqrt{3k}}$;
 (xxv) $f(x) = x^3 + x^2$;
 (xxvi) $f(x) = 3x - \frac{2}{x}$;
 (xxvii) $f(x) = x^3 - \frac{1}{x^3}$;
 (xxviii) $g(y) = 4y^2 + 5y^5 - \frac{2}{y^2}$;
 (xxix) $x(t) = t^2 + 4t + 4$;
 (xxx) $s(t) = (t + 2)^2$.

7. Find the equation of the tangent to the graph of $f(x)$ at x_0 .
- (a) $f(x) = x^2 + 2x + 3$, $x_0 = -1$;
 (b) $f(x) = \frac{1}{3}[x(\sqrt{x} - 1) - x^3]$, $x_0 = 4$;
 (c) $f(x) = \frac{x^3 - \frac{1}{\sqrt{x}} + 2}{\sqrt{x}}$, $x_0 = 3.1$.
8. A particle is moving along the x axis in such a way that its position (in cm) after t seconds is given by the formula $x(t) = 3 + 2t - \frac{1}{2}t^2$.
- (a) Find the initial position of the particle (i.e., the position in the moment $t = 0$) and its position after 3 s.
 (b) At which moment will the particle reach the origin of the coordinate system?
 (c) Find the formula for the velocity v after t seconds. [*Hint*: differentiate $x(t)$.]
 (d) What is the initial velocity? What is the velocity after 3 s? Does your result make sense?
 (e) Prove that the motion is uniformly retarded, i.e., the acceleration is negative and constant.

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