Derivatives in a nutshell

The *derivative* of a function of the form $f(x) = ax^p$ is the function $f'(x) = pax^{p-1}$. Example: if $f(x) = 3x^4$, then $a = 3$, $p = 4$, so $f'(x) = 4 \cdot 3x^{4-1} = 12x^3$. Notice that p may be zero (like in $f(x) = 5$); it can also be a negative number (like in $f(x) = \frac{3}{x^2} = 3x^{-2}$), a fraction (like in $f(x) = -2x\sqrt{x} = -2x^{\frac{3}{2}}$), or even both (like in $f(x) = \frac{7}{\sqrt[3]{x^2}} = 7x^{-\frac{2}{3}}$).

The process of finding the derivative of a function is called *differentiation*.

The derivative of the sum of two (or more) functions is the sum of their derivatives (and likewise for difference of functions). Examples: (i) if $f(x) = 2x^3 + 3x^2$, then we have $(2x^3)' = 6x^2$ and $(3x^2)' = 6x$, so $f'(x) = 6x^2 + 6x$. (ii) If $f(x) = x - \sqrt{x}$, then we have $x' = 1$ and $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$, so $f'(x) = 1 - \frac{1}{2\sqrt{x}}$.

Knowing how to find the *derivative function* of $f(x)$, you can also compute the *derivative of the function $f(x)$ at some point x_0*, i.e., the value $f'(x_0)$. Example: in order to find the derivative of the function $f(x) = 7x^2 + 2\sqrt{x}$ at the point $x_0 = 2$, we first compute $f'(x) = 14x + 1/\sqrt{x}$ and then substitute 2 for x, obtaining $f'(2) = 14 \cdot 2 + 1/\sqrt{2} \approx 28.7$.

Since the derivative of a function $f(x)$ with respect to x (i.e., treating x—as usually—as the independent variable) measures the (*instantaneous*) *rate of change* of $f(x)$ w.r.t. x, it can be applied to, e.g., physics. For example, if $x(t)$ is the *displacement function*, then $v(t) = x'(t)$ is velocity and $a(t) = v'(t)$ is in turn acceleration.

To be continued...